Avrupa Bilim ve Teknoloji Dergisi
Özel Sayı 29, S. 46-51, Aralık 2021

Araştırma Makalesi

www.ejosat.com ISSN:2148-2683

European Journal of Science and Technology
Special Issue 29, pp. 46-51, December 2021

Research Article

Enhanced Sparse Representations of Spike Waveforms Obtained by using the Basis Pursuit Approach

Görkem Serbes[1*]

[1] Yıldız Teknik Üniversitesi, Elektrik-Elektronik Fakültesi, Biyomedikal Mühendisliği Bölümü, İstanbul, Türkiye (ORCID: 0000-0003-4591-7368), gserbes@yildiz.edu.tr

(International Symposium on Multidisciplinary Studies and Innovative Technologies (ISMSIT) 2021 – 21-23 October 2021)

(**DOI:** 10.31590/ejosat.1009464)

ATIF/REFERENCE: Serbes, G. (2020). Enhanced Sparse Representations of Spike Waveforms Obtained by using the Basis Pursuit Approach. *European Journal of Science and Technology*, (29), 46-51.

Abstract

In the extracellular neural recordings, the spike waveforms formed by the neurons nearby the recording electrode must be sorted according to their morphology. This process is called as spike sorting and it is an important prerequisite in neural decoding algorithms. Low Q-factor wavelet transforms are frequently being used as feature extractors to detect the discriminative patterns between adjacent neurons' activity. However, the wavelet coefficients are highly sensitive to noise that may occur due to the employed instrumentation system and the local field potentials defined as the total activity of nearby neurons. However, enhanced sparse representations of the spike wave forms, having reduced noise activity, can be attained by using the basis pursuit method that is applied to the tunable Q-factor wavelet transform coefficients. In the tunable Q-factor wavelet transform, the Q-factor of the wavelet filters can be tuned according to the signal of interest with a controllable redundancy. In the proposed study, enhanced sparse representations of the spike waveforms were obtained by using the basis pursuit approach. Later, the energy values of the decomposed subbands were employed as features that can discriminate morphological differences in spike shapes. Finally, the obtained features were fed to k-nearest neighbors and decision trees learning models in an unbiased cross-validation scheme to objectively measure the effect of the enhanced sparsity decomposition. The qualitative and quantitative results show that the enhanced sparsity-based energy features are superior to the traditional low Q-factor based wavelet decomposition in terms of the accuracy metric.

Keywords: Spike sorting, the tunable Q-factor wavelet transform, sparsity, classification.

İğnecik Dalga Biçimlerinin İyileştirilmiş Seyrek Temsillerinin Temel Takip Yaklaşımı Kullanılarak Elde Edilmesi

Özet

Hücre dışı sinirsel kayıtlarda, kullanılan elektrodun yakınındaki sinir hücrelerinin oluşturduğu iğnecik dalga biçimlerinin morfolojilerine göre sıralanması gerekmektedir. Bu işleme iğnecik sıralaması adı verilir ve sinirsel kod çözme algoritmalarında kullanılan önemli bir ön koşuldur. Düşük Q-faktörlü dalgacık dönüşümleri, birbirine yakın sinir hücrelerinin aktiviteleri arasındaki ayırt edici örüntüleri tespit etmek için öznitelik çıkarıcılar olarak sıklıkla kullanılmaktadır. Fakat, farklı alt bantlardaki dalgacık katsayıları, kullanılan enstrümantasyon sistemi nedeniyle oluşan gürültü bileşenlerine ve yakındaki sinir hücrelerinin toplam aktivitesi olarak tanımlanan yerel alan potansiyellerine oldukça duyarlıdır. Bununla birlikte, azaltılmış gürültü aktivitesine sahip ğnecik dalga biçimlerinin geliştirilmiş seyrek temsilleri, ayarlanabilir Q faktörü dalgacık dönüşümü tabanlı katsayılara uygulanan temel takip yöntemi kullanılarak elde edilebilir. Ayarlanabilir Q faktörü dalgacık dönüşümünde, dalgacık filtrelerinin Q faktörü, kontrol edilebilir bir fazlalık ile ilgili sinyale göre ayarlanabilir. Önerilen çalışmada, dalgacık katsayılarına uygulanan temel takip yaklaşımı kullanılarak iğneciklerin iyileştirilmiş bir seyrek gösterimi elde edilmiştir. Daha sonra, ayrışmış alt bantların enerji değerleri, iğnecik şekillerindeki morfolojik farklılıkları ayırt edebilen özellikler olarak kullanılmıştır. Son olarak, elde edilen öznitelikler, geliştirilmiş seyreklik ayrıştırmasının etkisini nesnel olarak ölçmek için tarafsız bir çapraz doğrulama şemasında k-en yakın komşular ve karar ağaçları öğrenme modellerine beslenmiştir. Niteliksel ve niceliksel sonuçlar, iyileştirilmiş seyreklik tabanlı enerji özelliklerinin, doğruluk metriği açısından geleneksel düşük Q faktörüne dayalı dalgacık ayrıştırmasından daha üstün olduğunu göstermektedir.

Anahtar Kelimeler: İğnecik sıralama, ayarlanabilir Q-faktörü dalgacık dönüşümü, seyreklik, sınıflandırma.

[*] Corresponding Author: Yıldız Teknik Üniversitesi, Elektrik-Elektronik Fakültesi, Biyomedikal Mühendisliği Bölümü, İstanbul, Türkiye, ORCID: 0000-0003-4591-7368, gserbes@yildiz.edu.tr

1. Introduction

Extracellular neural activity recordings are frequently employed in clinical and scientific studies such as in decoding algorithms used for extracting information from the activity of neuronal populations. Mostly, these recordings are interpreted as point processes, and a spike detection algorithm is needed to predict the locations at which action potentials occurred. Spike detection is a very important prerequisite to be able to correctly analysis spike trains because the success of localizing the occurrence of individual spikes would critically affect the performance of all following analysis steps.

In extracellular neural recordings, the obtained signals normally consist of spikes created from multiple adjacent neurons (from unknown number of neurons). To obtain the response of each neuron, the detected possible spike events must be sorted according to their time-behavior to assign each activity to a separate neuron. This operation is named as spike sorting and it is a gold standard in neuroscience. With the progresses in technology, the usage of multi-electrode arrays becomes more popular in neuroscience [1]. When the number of recording electrodes increases in a system, the need for an automatic sorting approach also raises as human controlled sorting becomes a hard and time-consuming task. The shapes of spikes may change due to various affects such as the morphology of its dendritic tree and the distance/orientation of related spike relative to the recording location [2]. The difficulty of spike sorting can be affected from multiple factors. It was shown that the time-amplitude behavior of spikes for a specific neuron can vary; for example, the amplitude of a spike waveform can decrease by up to 80% in the course of a complex spike burst [3]. The overlapping spikes phenomenon, which takes place when two or more nearby neurons fire action potentials simultaneously, creates another complication in spike sorting process. Moreover, the recording electrodes may move faintly in the brain tissue because of the external physical constraints during the recording and this may cause variations in the spike waveform [4].

In spike sorting the most important step is extracting features that have the capability of differentiate the spike waveforms produced by various neurons. Later these features are given to a clustering method to group the spikes having similar features. In literature various feature extracting and spike sorting methods were proposed. For instance, a spike sorting approach that uses the peak to peak amplitude and width of the spikes as features was proposed in [5]. The principal component analysis was used for feature extraction in [6]. A spike labelling approach that uses template matching was employed in [7]. The k-means clustering and wavelet transform (WT) methods were employed together in [8]. An approach using WT based features grouped by superparamagnetic clustering was used in [9]. The k-means clustering and independent component analysis (ICA) methods were utilized together in [10]. Additionally, in [11-12] discrete wavelet transform (DWT) coefficients were used as discriminative features in spike sorting algorithms, but the WTs employed in these manuscripts were using constant Q-factor filters.

In literature, the DWT was frequently used as a robust feature extractor in spike sorting algorithms. By employing the time localization ability of wavelet-based features, small changes in the morphology of spikes that are very localized in time can be discerned. In addition, the time-amplitude behavior information of the spikes can be represented by using many wavelet coefficients in contrast to the principal component analysis (PCA), in which most amount of the details about the spike waveforms is preserved only by the first 3 principal components. The more homogeneous distribution of information obtained by WT provides a better representation for cluster identification. In line with these considerations, a superior performance of wavelet usage, in comparison with PCA, was given for several simulated spike trains constructed under various noise levels in [9]. However, in previous attempts, the used DWTs had a common property; they all used constant Q-factor filters. In a WT, the Q-factor of wavelet indicates the time behavior of the mother wavelet. Morphologically spikes have transient characteristics and to obtain features with more discriminative power, they must be decomposed with a DWT which has the capability of changing its Q-factor continuously. By doing this, for analyzed spike, an optimum Q factor can be found which tunes the transient behavior of that spike best and a sparse representation of these spikes can be achieved. Later these sparse coefficients can be used in different clustering methods for evaluating their discriminative capability.

Various WTs, having the ability to tune their employed wavelets, have been utilized in processing transient and oscillatory behavior biomedical signals. The tunable Q-factor wavelet transform (TQWT) [13] is a totally discrete WT for which the Q-factor of the underlying wavelet and the asymptotic redundancy (over-sampling rate) of the transform can be easily and independently specified. Therefore, by tuning the Q-factor, the oscillatory/transient behavior of the wavelets can be selected to match the oscillatory/transient behavior of the signal of interest, so as to magnify the sparsity of a sparse signal representation. The TQWT was successfully employed in resonance-based decomposition of lung sounds that aims to separate wheeze, crackle and vesicular sounds into three individual channels [14]. In [14], the Q-factor and over-sampling rate parameters of the TQWT were determined in a way that the wavelet filters match the morphology of the crackles and wheezes. After the determination of the optimal Q-factor (low Q-factor), the crackles, which have similar time-domain characteristics with spikes, were successfully separated from the wheezes and background noise. Additionally, the TQWT was employed to extract embolic signal information from background activity and artifacts [15]. In [15], the resonance-based decomposition, in which the TQWT was employed to catch the morphology of signal interest, was applied to Doppler ultrasound signals and quasi-periodic embolic signals were obtained (decomposed into the high Q-factor channel). In the proposed study, we intend to employ TQWT as a feature extractor for spike sorting problem due to its tunable Q-factor property. The sparsity ability of the TQWT will be used to obtain the wavelet coefficients having high discriminative capability, especially in high noise environments. The following sections of the study are organized as follows; Section 2 gives information about materials and methods. Section 3 gives the experimental results and finally, Section 4 presents the conclusion and discussion.

2. Material and Method

2.1. Dataset Information

In spike detection and spike sorting studies, a critical issue is the existence of a dataset in which the exact locations of the spikes and the spike labels are defined. In [9], such a synthetic dataset that consists of simulated brain activity signals was proposed. In this dataset, the simulated brain signals were composed by employing a database that consists of 594 various spike waveforms derived from the recordings of neocortex and basal ganglia. To mimic brain activity in a realistic manner, a background noise was generated by randomly selecting spikes from 594 waveforms and the selected ones were superimposed at random amplitudes and times. In each recording three different spike waveforms, having a normalized peak amplitude, were placed into the background activity resulting in a spike train. The normalized peak amplitude of each spike was set to 1 and the background noise activity level was discovered from its standard deviation relative to the spike peaks. The employed noise standard deviation values were set as 0.05, 0.1, 0.15, and 0.2 for all the recordings. Additionally, more noise levels set to 0.25, 0.30, 0.35, and 0.4 were defined for just one case in which relatively easy to differentiate spike waveforms were employed. In our proposed approach, this recording having 8 noise levels was chosen as the ground truth dataset to be able to measure the performance of TQWT based sparse representations of spikes. The sampling rate of the recording is 24 kHz and the three distinct spike waveforms were located by using a Poisson distribution of interspike intervals having a mean firing rate of 20 Hz. The refractory period between the spike shapes belonging to the same class was chosen as 2mS.

2.2. The Usage of Tunable Q-factor Wavelet Transform in Feature Extraction

2.2.1. The Tunable Q-factor Wavelet Transform

The constant-Q synthesis and analysis, in which a group of band-pass filters having same Q-factors are employed, has been successfully used in the analysis of non-stationary biomedical signals [16-18]. For a band-pass filter, the ratio of that filter's center frequency to its bandwidth is referred as its Q-factor. When the temporal response of constant-Q filters is investigated, it is seen that a more sustained oscillatory behavior exists when high Q-factor filters are used. In contrast, a more transient time behavior is seen in the temporal response of low Q-factor filters. Classical dyadic DWT is a constant Q-factor transform having low Q-factor characteristics and it provides an efficient representation of piecewise smooth signals such as the spikes [9]. However, the energy distribution between the scales is highly sensitive to the amount of noise that is superimposed onto the signal of interest. Therefore, transforms that can be employed to obtain the sparse representations of the biomedical patterns, in which the noise effects are minimized, are highly needed. In the extracellular neural recordings, the spike waveform morphology is corrupted by the instrumental noise and the local field potentials (the total activity of nearby neurons). Hence, in the proposed study we have intended to use WTs that can enhance the sparsity of spike representations to be able to extract the differentiation in the spike shapes created by different neurons more efficiently and to be able to reduce the disruptive effect of noise.

The TQWT is an overcomplete wavelet transform, in which the Q-factor of the analysis/synthesis filters can be easily and continuously adjustable. By adjusting the Q-factor of employed filters, the time-domain behavior of the signal of interest can be represented more efficiently and sparse spike representations can be obtained. It is expected that the enhanced sparsity should in turn improve the performance of sparsity-based feature extraction for spike sorting in the extracellular neural recordings under high noise. In the TQWT, mainly two parameters are used to control transform properties: i) the Q-factor (Q) that controls the oscillatory behavior of the time-domain response of wavelet filters, and ii) the over-sampling rate (r) that controls the redundancy level of applied transform. The r value is measuring the overlap ratio between the consequent band-pass filters' content in the frequency axis. When the Q value is kept constant and the r value is increased, the overlapping parts of the band-pass filter frequency responses, having same Q-factors, will increase too. More details about the theoretical explanation about the TQWT can be found in [13].

2.2.2. Proposed Tunable Q-factor Wavelet Transform based Sparse Representations of Spikes

The TQWT is an over-complete transform therefore, its time-scale representation is not unique - other sets of wavelet coefficients (different decompositions), having perfect reconstruction property, can be acquired that also represent a given signal exactly. An enhanced sparsity in the sparse representations of signal of interests can be achieved by using the basis pursuit (BP) method. BP is a principle for decomposing a signal into an "optimal" superposition of dictionary elements, where optimal refers to having the smallest ℓ_1 norm of coefficients among all such decompositions [19]. In our proposed approach, we have applied the BP method to obtain the enhanced sparse representations of TQWT coefficients that were extracted from spike waveforms. Due to the piecewise smooth time-domain and band-limited frequency-domain characteristics of spike signals, the Q-factor and redundancy parameters were set to 1 and 3 respectively. After 5 level decomposition, energy values of each 5 detail and the last approximation subbands were calculated as features resulting in a 6-element row vector. After obtaining the feature vectors, even the spike sorting is a clustering problem due to its nature, a three-class discrimination problem was carried-out by using k-nearest neighbors (k-NN) and decision tree (DT) learning models. The reason behind handling the spike sorting as a classification problem is to be able to objectively assess the degree of contribution obtained when the enhanced sparse representations of spike coefficients were employed.

Before feeding the spike waveforms into feature extraction and classification modules, each spike shape was extracted from the long recording signal at the first place. Later, 64 time-points segments samples were created in which the maximums of each spike were aligned to the same time instant as advertised in [9]. Only the non-overlapping spikes were utilized in the analysis for an objective comparison of feature extraction methods. The created dataset, in which each sample consists of one spike waveform having 64 time-points long, was partitioned into two sub-sets, named as train and test sets, for a fair validation of performance comparison. The samples in the train and test sets were randomly selected by following the holdout approach resulting in 80% and 20% sample distributions respectively. To avoid from the possible overfitting situation in learned models, the 80% train data was reorganized by using K-Fold cross validation with $K = 5$. The flow-chart of the proposed approach was given in Figure 1 for further understanding.

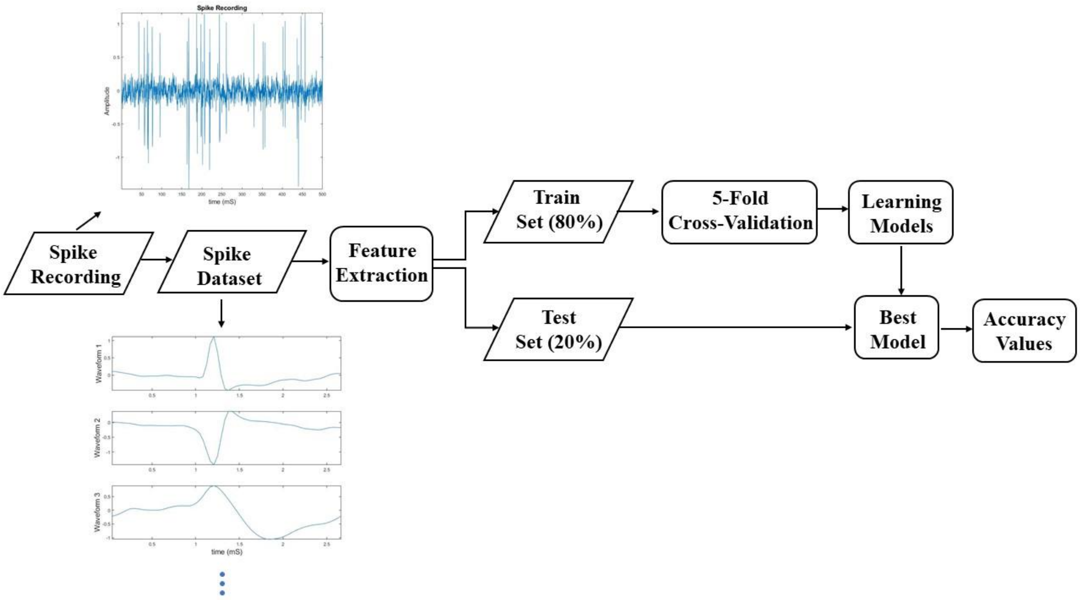

Figure 1. The flowchart of the proposed approach.

3. Results and Discussion

3.1. Qualitative Results

The main contribution of our study is using wavelet filters that have tunable time and frequency domain characteristics in accordance with the spike waveform morphology. To do so, proper Q and r parameters were chosen as 1 and 3 respectively. Additionally, the BP algorithm was employed to obtain an enhanced sparse representation of spike waveforms in the wavelet coefficient domain. It is seen that the spike information that was attained in the wavelet coefficients can be represented in a very sparse way in decomposed subbands. As it is shown in the right-side of Figure 2, most of the noise components were minimized while the components belong to spike activity are preserved in subbands 2, 3 and 4. This sample was taken from the extracellular recording whose noise level is 0.1. The energy percentages of each subband with respect to total signal energy are given in the right side of each subplot. When the left-side plot is investigated, it is seen that the energy of spike waveform can not be localized, and a homogenous energy distribution is seen over all subbands. Whereas most of the spike energy can be successfully localized in the second band as it is shown in the right-side plot. This sparse representation ability of the TQWT over spike waveforms has directed us to use energy values obtained from the decomposed subbands as features in spike shape discrimination problem.

3.1. Quantitative Results

The mean and standard-deviation of accuracy values obtained by using 5-fold outputs are given in Tables 1 and 2 under the “Validation Set” section for the cases in which the enhanced sparsity is not-used and used respectively. For the realistic noise levels higher than 0.10, the accuracy values obtained by using enhanced sparsity were always higher than the non-sparsity used cases for both k-NN and DT. Additionally, the standard-deviation values obtained with the enhanced sparsity are relatively lower than the non-sparsity case for both classifiers. This shows us that the features obtained by using the enhanced sparsity are more robust compared with the features extracted from non-sparsity decomposition. In the test set results, it is seen that accuracy values up to 98% were obtained by using the enhanced sparsity-based features even for very high noise values for both k-NN and DT classifiers. For the highest two noise levels (0.35 and 0.40), the accuracy increments obtained by using enhanced sparsity were approximately 25% and 10% when the k-NN and DT classifiers were employed respectively. The detailed representation of obtained accuracy values for both the validation and test sets by using k-NN and DT classifiers are listed in Tables 1 and 2.

4. Conclusions and Discussion

In the proposed study the TQWT was used as a feature extractor for the extracellular spike recordings. Traditionally, low Q-factor DWTs are employed for extracting discriminative information between various neuron activity in the spike sorting problem. However, the pursued discriminative information was mostly lost when high noise components have been superimposed on spike activity. An enhanced sparsity property can be reached by using the BP approach that is applied on TQWT based time-scale coefficients and the amplitude of the noise components can be minimized. In our proposed study, a data-set consists of various spike shapes under different noise levels was processed in a scenario that the enhanced sparsity was obtained and not-obtained by using the BP approach. The extracted sparse features were fed to k-NN and DT classifiers to be able to measure the performance of enhanced sparsity in spike morphology analysis. The qualitative and quantitative results show that a significant improvement in spike shape identification was achieved when the wavelet energy features obtained by enhanced sparse representations were employed. During the analysis, only non-overlapping spikes were utilized for an objective comparison, however, we are planning to apply same enhanced sparsity-based feature extraction approach to overlapping spikes in a future study.

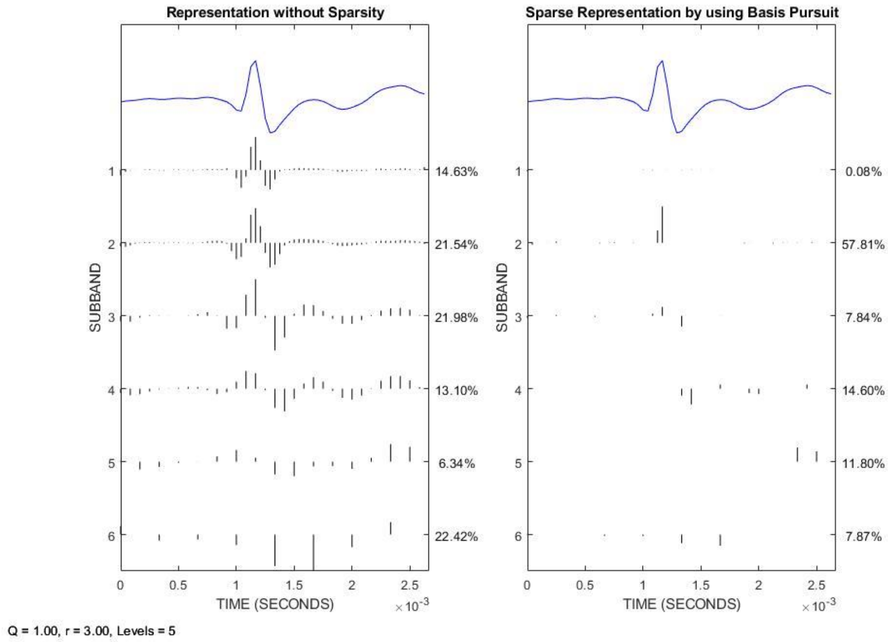

Figure 2. Representation of a spike signal in time-scale domain when the BP approach is applied (right-side) not applied (left-side).

Table 1. The accuracy values obtained when the enhanced sparsity is not used. "Average" and "std"stands for the mean and standard-deviation accuracy values of all folds that are used in 5-fold cross-validation strategy.

Without Sparsity						
	Validation Set				**Test Set**	
	k-NN		**DT**		**k-NN**	**DT**
Noise Level	**average**	**std**	**average**	**std**		
0.05	100.00	0.00	100.00	0.00	100.00	100.00
0.10	99.73	0.25	100.00	0.00	99.82	100.00
0.15	98.42	0.86	99.77	0.23	98.51	100.00
0.20	95.29	1.01	99.07	0.68	94.95	97.76
0.25	88.40	1.92	96.33	1.04	89.17	97.10
0.30	81.99	2.16	94.63	1.00	84.95	94.10
0.35	76.41	2.00	90.66	2.51	72.41	89.07
0.40	70.93	2.32	86.58	0.74	70.51	88.09

Table 2. The accuracy values obtained when the enhanced sparsity is used. "Average" and "std"stands for the mean and standard-deviation accuracy values of all folds that are used in 5-fold cross-validation strategy.

With Sparsity						
	Validation Set				**Test Set**	
	k-NN		**DT**		**k-NN**	**DT**
Noise Level	**average**	**std**	**average**	**std**		
0.05	100.00	0.00	100.00	0.00	100.00	100.00
0.10	100.00	0.00	100.00	0.00	100.00	100.00
0.15	100.00	0.00	100.00	0.00	100.00	100.00
0.20	99.81	0.10	99.95	0.10	100.00	99.63
0.25	99.42	0.56	99.52	0.38	98.84	99.61
0.30	98.38	0.80	98.91	0.55	99.24	99.24
0.35	97.09	0.35	99.03	0.38	97.96	99.44
0.40	95.32	0.95	97.50	0.68	96.03	98.49

References

[1] Rizk, M., Bossetti, C. A., Jochum, T. A., Callender, S. H., Nicolelis, M. A., Turner, D. A., & Wolf, P. D. (2009). A fully implantable 96-channel neural data acquisition system. *Journal of neural engineering*, 6(2), 026002.

[2] Lewicki, M. S. (1998). A review of methods for spike sorting: the detection and classification of neural action potentials. *Network: Computation in Neural Systems*, 9(4), R53.

[3] Buzsáki, G. (2004). Large-scale recording of neuronal ensembles. *Nature neuroscience*, 7(5), 446-451.

[4] Chah, E., Hok, V., Della-Chiesa, A., Miller, J. J. H., O'Mara, S. M., & Reilly, R. B. (2011). Automated spike sorting algorithm based on Laplacian eigenmaps and k-means clustering. *Journal of neural engineering*, 8(1), 016006.

[5] Abeles, M., & Goldstein, M. H. (1977). Multispike train analysis. *Proceedings of the IEEE*, 65(5), 762-773.

[6] Harris, K. D., Henze, D. A., Csicsvari, J., Hirase, H., & Buzsaki, G. (2000). Accuracy of tetrode spike separation as determined by simultaneous intracellular and extracellular measurements. *Journal of neurophysiology*, 84(1), 401-414.

[7] Thakur, P. H., Lu, H., Hsiao, S. S., & Johnson, K. O. (2007). Automated optimal detection and classification of neural action potentials in extra-cellular recordings. *Journal of Neuroscience Methods*, 162(1-2), 364-376.

[8] Hulata, E., Segev, R., & Ben-Jacob, E. (2002). A method for spike sorting and detection based on wavelet packets and Shannon's mutual information. *Journal of neuroscience methods*, 117(1), 1-12.

[9] Quiroga, R. Q., Nadasdy, Z., & Ben-Shaul, Y. (2004). Unsupervised spike detection and sorting with wavelets and superparamagnetic clustering. *Neural computation*, 16(8), 1661-1687.

[10] Takahashi, S., Anzai, Y., & Sakurai, Y. (2003). A new approach to spike sorting for multi-neuronal activities recorded with a tetrode—how ICA can be practical. *Neuroscience research*, 46(3), 265-272.

[11] Geng, X., Hu, G., & Tian, X. (2010). Neural spike sorting using mathematical morphology, multiwavelets transform and hierarchical clustering. *Neurocomputing*, 73(4-6), 707-715.

[12] Geng, X., & Hu, G. (2012). Unsupervised feature selection by kernel density estimation in wavelet-based spike sorting. *Biomedical Signal Processing and Control*, 7(2), 112-117.

[13] Selesnick, I. W. (2011). Wavelet transform with tunable Q-factor. *IEEE transactions on signal processing*, 59(8), 3560-3575.

[14] Ulukaya, S., Serbes, G., & Kahya, Y. P. (2016, August). Resonance based respiratory sound decomposition aiming at localization of crackles in noisy measurements. In *2016 38th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)* (pp. 3688-3691). IEEE.

[15] Serbes, G., & Aydin, N. (2017, May). Resonance based pre-processing method for eliminating artifacts in Doppler ultrasound signals. *In 2017 25th Signal Processing and Communications Applications Conference (SIU)* (pp. 1-4). IEEE.

[16] Serbes, G., Aydin, N., & Gulcur, H. O. (2013, July). Directional dual-tree complex wavelet packet transform. *In 2013 35th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)* (pp. 3046-3049). IEEE.

[17] Serbes, G., & Aydin, N. (2012, January). Embolic Doppler ultrasound signal detection using modified dual tree complex wavelet transform. *In Proceedings of 2012 IEEE-EMBS International Conference on Biomedical and Health Informatics (pp. 945-947). IEEE.*

[18] Serbes, G., Sakar, C. O., Kahya, Y. P., & Aydin, N. (2011, September). Effect of different window and wavelet types on the performance of a novel crackle detection algorithm. *In International Conference on Hybrid Information Technology* (pp. 575-581). Springer, Berlin, Heidelberg.

[19] Chen, S. S., Donoho, D. L., & Saunders, M. A. (2001). Atomic decomposition by basis pursuit. SIAM review, 43(1), 129-159.